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Weight Comparison of Ring- vs Ring/Stringer-Stiffened Cylindrical Pressure Hulls

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Optimum ring-stiffened cylindrical pressure hulls have slightly lower structural efficiency than optimum ring/stringer-stiffened cylindrical pressure hulls for a constant ocean collapse depth. However, efficiency is achieved in ring/stringer-stiffened hulls with thinner walls which result in significantly higher wall stresses. Example steel and titanium hull designs are presented for collapse depths ranging from 2000 to 7000 ft. Titanium is superior in elastically stressed designs but alloys in either steel or titanium with sufficiently high yield strengths after welding are not available to develop the maximum potential structural efficiency of either configuration at deeper depths. A nondimensional structural optimization analysis which considers the several possible modes of instability and recognizes a number of practical design restraints (for example, a maximum ring/stringer height) is used to obtain these results.

Nomenclature†

B = $m\pi R/L_1$
D_f = $Et^3/12(1 - \nu^2)$
D_x, D_y, D_{33} = coefficients of flexural and torsional rigidity for orthotropic shells
E = Young's modulus
I_x = moment of inertia per unit of circumferential width of cylinder wall cross section taken about the centroid of the composite cross section
I_y = moment of inertia per unit of length of cylinder wall cross section taken about the centroid of the composite cross section
J_x = stringer torsion constant per unit of circumferential width of cylinder wall
J_y = ring torsion constant per unit of length of cylinder wall
L = length of cylindrical hull or hull compartment
L_1 = length; equal to d_y for panel instability calculations, equal to L for over-all instability calculations
m = number of axial half-waves
n = number of circumferential full waves
p = hydrostatic buckling pressure, psi
R = cylinder radius, measured to midplane of hull wall
W_i = weight of hull/unit of surface area
Y = effective width correction factor
Z = $(1 - \nu^2)^{0.5}(R/t)(L/R)^2$
β = buoyancy factor; hull weight/weight of displaced sea water (in this paper, hull weight is based on a hull with $L/R = 30$ plus hemispherical end closures but without internal bulkheads)
μ_1 = increase in effective cross-sectional area of the shell due to the stringers
μ_2 = increase in effective cross-sectional area of the shell due to the rings
ν = Poisson's ratio
ρ = material density
σ_y = stress in y or circumferential direction
χ_1 = change in extensional stiffness caused by the eccentricity of the stringers
χ_2 = change in extensional stiffness caused by the eccentricity of the rings

Subscripts

cr = critical
x or 1 = axial coordinate
y or 2 = circumferential coordinate
cl = classical

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† See Fig. 1 for notation for stiffened cylinder dimensions.

Introduction

RING-STIFFENED cylinders have been the primary structural configuration for pressure-hull design since 1898 when the first modern submarine torpedo boat "Holland" was launched.[1] The importance of structural weight in submersible vehicles has been recognized from the beginning, and, with the development of principles for efficient design, studies have been conducted to determine the most efficient distribution of material between the rings and the cylinder wall, for example, Refs. 2 and 3. However, more recent studies[4,5] have shown that lighter designs may result when the cylinder is stiffened with stringers as well as rings. These observations, in general, have been made with reference to aerospace vehicles which encounter low-to-moderate aerodynamic pressure levels.

The present paper reports on a study to determine if the use of both rings and stringers in hydrostatically compressed, cylindrical pressure hulls capable of ocean operations to a collapse depth of 10,000 ft can produce any significant benefits over similar hulls stiffened only with rings. The most direct benefit, of course, would be either lighter designs at a given collapse depth, or, alternately, greater collapse depth capability for a constant structural weight. But more indirect benefits may also exist; for example, ring/stringer-stiffened hulls may possess some features which facilitate fabrication, such as thinner hull walls. Whether any advantage can be shown for hulls stiffened with rings and stringers depends, as is well known, upon the mechanical properties of available materials, particularly the compressive yield strength. In this paper, only steel and titanium are considered. These two materials are appropriate choices for tactical or commercial vehicles operating at the depths of interest. Other materials, such as aluminum and glass may offer distinct advantages; however, they are presently in the developmental stage and are being considered primarily for research vehicle applications.

Analysis

The structural optimization analysis presented in Ref. 5 for cylinders stiffened with rings and/or stringers of rectangular cross section may be used in the present study if the proportion of lateral-to-axial load in the cylinder wall is adjusted to represent a uniform external hydrostatic pressure (2:1 ratio). This analysis is based upon the same criterion for efficient structural design utilized by Gerard[2] and Nickell and Crawford[3]; i.e., at least the two lowest buckling modes possible in the structure are critical under the applied loading. Oerard solves for the lightest design which meets this requirement, without regard to the practicality of the resulting design. It is tacitly assumed that if the resulting gages are nonstandard or the ring spacing is too small, slight modifications to the design to correct these deficiencies can be made, using trial-and-error techniques, with only a small increase in weight. This type of analysis has been termed a minimum weight analysis. Nickell and Crawford also solve for the lightest design but they introduce a design restraint, namely, a specified number of rings. By repeating their analysis for a range of number of rings, results are obtained which show the effect on weight of compromising on the optimum number of rings for reasons of economy in fabrication, for example. In order to differentiate between the two analytical approaches, the type of analysis performed by Nickell and Crawford has been referred to as an optimization analysis. The optimization analysis of Ref. 5, when applied to hulls stiffened only with rings, is similar to that of Ref. 3; however, subsequent advances in the theory of stability of stiffened shells have been incorporated into Ref. 5, and other design restraints may be specified if desired. In particular, the equations derived by Baruch and Singer[6] for defining the classical hydrostatic buckling pressure for stiffened cylinders are utilized; these equations take into account the effects of ring/stringer eccentricity relative to the cylinder wall midplane:

$$\frac{p_{cl}}{E} = \frac{F}{[(0.5B^2 + n^2)(R/t)^3 12(1-\nu^2)]} \tag{1}$$

where

$$F = \left(\frac{D_x}{D_f}\right)B^4 + \left(\frac{D_y}{D_f}\right)n^4 + \left[\left(\frac{2D_{33}}{D_f}\right) - 1 + 2\nu\right]2B^2n^2 + 12\left(\frac{R}{t}\right)^2\left[\chi_1(-B^3a_n) + \chi_2(-2n^2 - b_nn^3) + (1+\mu_2)(1+b_nn) + \nu Ba_n + \frac{\chi_2{}^2n^4}{[(1-\nu^2)Y + \mu_2]} + \frac{\chi_1{}^2B^4}{(1-\nu^2+\mu_1)}\right] \tag{2}$$

$$a_n = D_{1n}/D_{0n} \tag{3}$$

$$b_n = D_{2n}/D_{0n} \tag{4}$$

$$D_{0n} = 0.5(1-\nu)(1+\mu_2)n^4 + [(1+\mu_1)(1+\mu_2) - \nu]B^2n^2 + 0.5(1+\mu_1)(1-\nu)B^4 \tag{5}$$

$$D_{1n} = -0.5(1+\nu)\chi_2Bn^4 + (1+\mu_2)[\chi_1B^3 + 0.5(1-\nu)B]n^2 + 0.5\chi_1(1-\nu)B^5 - 0.5\nu(1-\nu)B^3 \tag{6}$$

$$D_{2n} = 0.5(1-\nu)\chi_2n^5 + [(1+\mu_1)\chi_2B^2 - 0.5(1-\nu)(1+\mu_2)]n^3 + \{[0.5(1+\nu)\nu - (1+\mu_1)(1+\mu_2)]B^2 - 0.5(1+\nu)\chi_1B^4\}n \tag{7}$$

$$D_x/D_f = 12(1-\nu^2)I_x/t^3 \tag{8}$$

$$D_y/D_f = 12(1-\nu^2)I_y/t^3 \tag{9}$$

$$2D_{33}/D_f = 3(1-\nu)[(\tfrac{2}{3}) + (J_x/t^3) + (J_y/t^3)] \tag{10}$$

$$\chi_1 = 0.5\mu_1[1 + (b_x/t_1 \cdot t_1/t)]t/R \tag{11}$$

$$\chi_2 = 0.5\mu_2[1 + (b_y/t_2 \cdot t_2/t)]t/R \tag{12}$$

$$\mu_1 = (1-\nu^2)(b_x/t_1)(t_1/t)^2 t/d_x \tag{13}$$

$$\mu_2 = (1-\nu^2)(b_y/t_2)(t_2/t)^2 t/d_y \tag{14}$$

It should be pointed out that in Eqs. (1–14) the rings and stringers are treated as though they were distributed or smeared out. Singer and Haftka,[7] using equations for discretely stiffened shells developed by Baruch,[8] have shown that little error is incurred in ring-stiffened cylinders from such an approach provided that 1) the geometric parameter Z is small, 2) the ring eccentricity is high, and 3) the ring spacing is small with respect to the cylinder length. Results obtained from the present study show that optimum designs meet these three requirements at the ocean depths of interest here; however, off-optimum designs with fewer rings may be unconservative. Although comparisons of ring-stiffened cylinders in Ref. 8 show differences as high as 20%, differences of that magnitude are not expected from the present analysis since a wall effective width correction factor Y has been included in the computations for I_y[9] and elsewhere. This factor ranges as low as 0.5 for cylinders stiffened with widely spaced rings.

Three possible buckling modes are considered in this study 1) local buckling of the wall/stringer/ring elements, 2) panel buckling of the wall/stringer composite between rings and 3) over-all buckling of the wall/stringer/ ring composite between end bulkheads. Classical hydrostatic buckling pressures for modes 2 and 3 are found by minimizing p_{cl} in Eq. (1) with respect to the number of circumferential full waves n and the number of axial half-waves m; therefore both axisymmetric and asymmetric buckling are automatically taken into account. In cylinders stiffened only with rings, mode 1 relates only to the ring elements; classical hydrostatic buckling pressures for modes 2 and 3 are found from Eq. (1) by properly defining the elastic constants. Local wall buckling in ring- and stringer-stiffened cylinders is predicted by treating this element as a biaxially loaded flat plate Local buckling calculations are carried out for the ring and stringer elements after an analysis has been performed to determine loads acting on these elements. For details, see Ref 5.

Because there is some discrepancy between theory and the available experimental data for modes 2 and 3 (see Ref. 3 for example), the lower bound for the critical hydrostatic buckling pressure for design purposes is set here equal to 0.8 p_{cl}. Higher critical buckling pressures are permitted, however, if the reduction factor for the stiffened hull subjected to axial pressure alone exceeds 0.8. This reduction factor is based on the minimum axial postbuckling pressure, which may be considered to be a measure of the sensitivity of a given structure to initial geometric imperfections. For further discussion, see Ref. 5.

The practical restraints imposed upon the analysis and the nondimensional parameters used to describe them are: 1) wall thickness known, R/t; 2) maximum ring/stringer height specified, $(b_x/R)_{\max}$, $(b_y/R)_{\max}$; and 3) maximum ring/stringer slenderness ratio specified, $(b_x/t_1)_{\max}$, $(b_y/t_2)_{\max}$. The analysis proceeds in a manner similar to that described in Ref. 5. Briefly, the hydrostatic pressure related to a given collapse depth is determined; the hull material, cylindrical length L, and radius R are assumed known. Values are assigned to the various restraints, and the ring/stringer positions relative to the cylinder-wall midplane are specified. A ring spacing is assumed. The stringers are given an initial thickness, and the proportions of the hull axial cross section are determined subject to the restraints imposed, such that local instability in one or more elements of the cross section does not occur before the value of the applied hydrostatic pressure is reached. Calculations for panel instability are carried out, and, if the critical collapse pressure for panel instability is less than the applied hydrostatic pressure, the stringer thickness is incremented. The process is repeated until these two pressures are identical. A similar procedure is subsequently followed for sizing the rings against collapse due to over-all instability. The entire process is repeated for additional ring spacings to determine the spacing resulting in the lightest design. Also, it may be desirable to investigate several wall thicknesses t to insure that the optimum t has been selected. For hulls stiffened only with rings, the wall between rings is treated as a short monocoque shell with simply supported edges; because R, t, the material, and the applied hydrostatic pressure have been specified, the analysis is used to solve directly for the ring spacing d_y. Ring sizing for over-all instability follows in a manner parallel to that described previously.

Parametric Study Results

Results are presented for the stiffened hull configuration shown in Fig. 1, and for a hull stiffened only with rings of the same configuration. Note that the rings and stringers have been attached to the inside surface of the hull because this is the conventional placement and also because available studies on the effect of ring/stringer eccentricity upon efficiency, Ref. 5 for example, show that inside placement is often superior. In addition, when outside placement is better than inside placement, the difference in efficiency is usually small. The configuration shown in Fig. 1 does not lend itself particularly well to fabrication; i.e., attach flanges for either stringers or rings are usually preferred, and possibly outstanding flanges also. Nevertheless, a qualitative comparison of ring- vs ring/stringer-stiffened hulls is valid.

The values of the geometric restraints observed in the figures and tables which follow are: $L/R = 2.0$, $(b_x/t_1)_{\max} = (b_y/t_2)_{\max} = 15.0$, $(b_x/R)_{\max} = 0.05$, and $(b_y/R)_{\max} = 0.10$. Of course, results will vary with the evaluation of L/R; the value chosen here is intended to represent a typical vehicle compartment which is simply supported at each end by watertight bulkheads. Similar qualitative results can be expected for other near values of L/R.

The material properties for steel and titanium assumed for this study are listed in Table 1. Note that several allowable stresses in the y or circumferential direction are shown in order to provide a varying upper bound on the elastic analysis.

Table 1 Material properties used in this study

Material	E, psi $\times 10^6$	ρ, lb/in.^{3}	ν	σ_y, psi $\times 10^3$
Steel	29.	0.30	0.3	100, 150, 200, 300
Titanium	16.	0.16	0.3	100, 150, 200

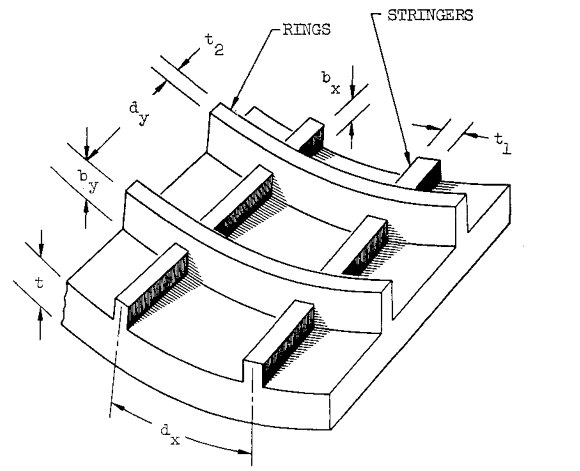

Fig. 1 View of hull section with internal rings and stringers of rectangular cross section, showing nomenclature for dimensions.

For preciseness in the analysis, σ_y should be considered the equivalent of the proportional limit stress, but for convenience it is considered here the equivalent of compressive yield stress. Steel and titanium sheet/plate with a compressive yield stress of 100,000 psi after welding (without subsequent heat treatment), and other desirable and necessary properties such as notch toughness, are readily available[10]; however, steel and titanium with higher strengths after welding are not generally in use in large pressure hulls although steel alloys with yield strengths to 350,000 psi have been announced.[11]

Figure 2 presents a comparison of optimum ring/stringer-stiffened hulls vs optimum hulls stiffened only with rings, where both configurations are fabricated from steel. The ordinate parameter represents weight per square inch of hull cylindrical surface area divided by cylinder radius in inches, whereas the abscissa scale shows ocean collapse depth in feet. The curve for monocoque cylinders has been obtained from the same analysis as the other curves. The R/t's have been selected to yield standard plate gages for a hull which is 30 ft in diameter; thus, standard gases may not necessarily result if the curves are used in connection with other hull diameters. (Note that if the hull is 30 ft in diameter, the maximum stringer-ring heights become $b_{x_{\max}} = 9.0$ in., $b_{y_{\max}} = 18.0$ in.). It is apparent that the lower envelopes for the rings-only configuration and the rings-and-stringers configuration are almost identical; therefore, optimum designs for these two configurations which satisfy the restraints imposed and lie on the lower envelope of the curves will weigh about the same. It is noteworthy, however, that the curves for cylinders stiffened only with rings are characterized by a rather sudden upward sweep to the right. This is caused by

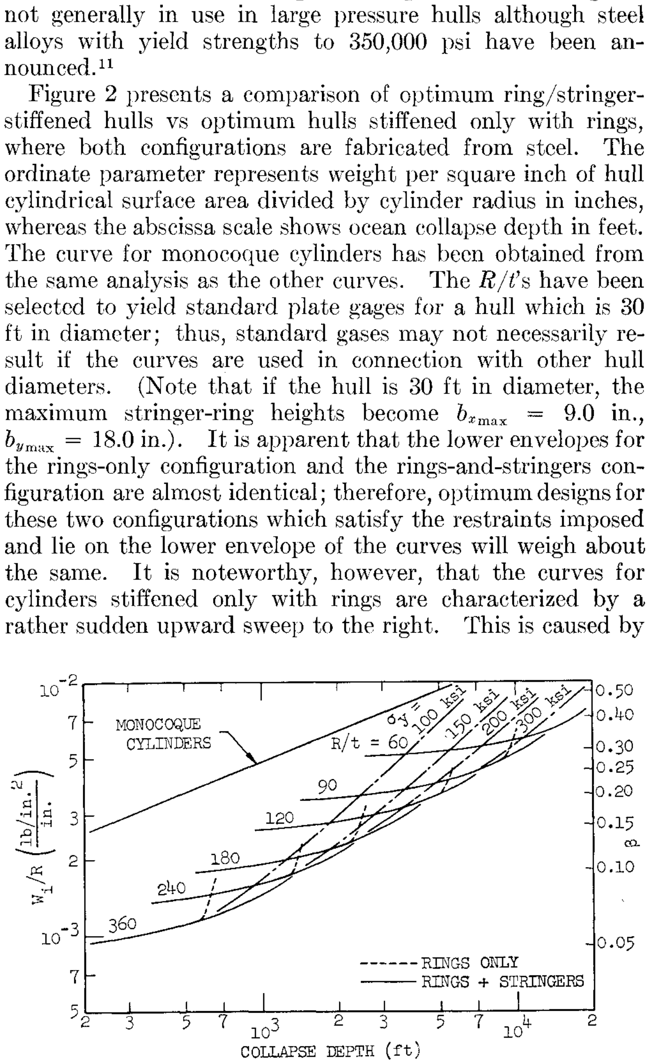

Fig. 2 Comparison of steel ring/stringer-vs ring-stiffened hulls.

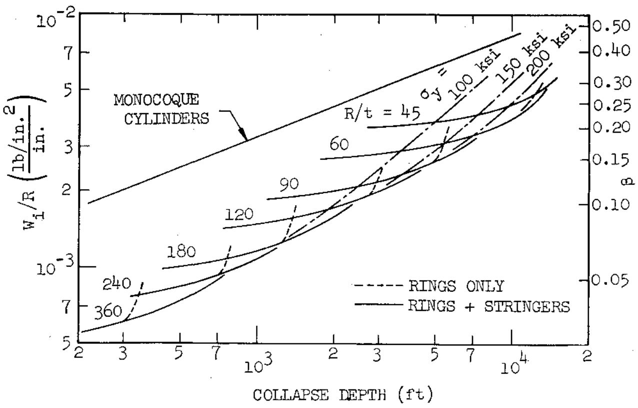

Fig. 3 Comparison of titanium ring/stringer-vs ring-stiffened hulls.

a change in the axial buckle pattern from $m = 1$ to $m \gg 1$. When this occurs, it is relatively inefficient to increase the circumferential stiffness of the hull through manipulation of the size and number of rings; an increase in hull wall thickness is preferable. The curves for cylinders stiffened with rings and stringers move upward to the right much more gradually, and sizable differences in the curves for a particular R/t favoring the ring/stringer configuration do occur at discrete points, particularly at collapse depths less than 4000 ft. The extent of these differences is a function of the number of R/t's considered, or to be more precise, the size of the increment in standard hull wall gages. In new designs, the quantity of material involved is generally sufficient to make special mill runs feasible for a specific hull wall thickness; thus, these differences may be avoided.

Four yield stresses for steel from Table 1 are superimposed on Fig. 2, where $\sigma_y = p \cdot (R/t)$; i.e., the allowable stress is compared with a circumferential stress calculated assuming the collapse pressure is carried by the hull wall alone. It is apparent that in order to realize the maximum potential structural efficiency inherent in either configuration, steels with yield strengths substantially in excess of 100,000 psi must be utilized whenever collapse depth exceeds approximately 1000 ft. This figure shows that steel with a yield strength of 200,000 psi is required for maximum structural efficiency in vehicles designed for collapse depths of approximately 4000 ft, whereas steel with a yield strength of 300,000 psi is required for maximum structural efficiency in vehicles designed for a collapse depth of 10,000 ft. It should be strongly emphasized here that these strength-depth relations (as well as subsequent ones cited for titanium) are those required to develop the maximum potential of the stiffened hull. Efficient hull designs can still be obtained with steels with substantially lower strengths, as pointed out by other authors,[12] provided that the monocoque hull for the same collapse depth is not also stress limited. Obviously, if a stiffened hull design is highly stress limited, then there is no difference in the weights of ring- and ring/stinger-stiffened designs. Stress limitation, it may be seen, represents an erosion of the basic advantage of a stiffened hull design. For example, at a collapse depth of 10,000 ft, structural weight increases about 60% if 150,000 psi yield steel is used instead of 300,000 psi yield steel. The effect of this increase in

Table 2 Sample elastically stressed steel hull designs at three ocean depths[a]

Configuration case		Rings only 1	2	3	Rings + stringers 4	5	6
a) Collapse depth = 2000 ft							
R/t		189.5	180	120	240	180	360
t	in.	0.95	1.00	1.50	0.75	1.00	0.50
σ_y	psi × 10^3	169	160	107	214	160	320
d_x	in.	...	...	...	10.27	20.46	4.95
b_x	in.	...	...	...	1.82	1.33	1.72
t_1	in.	...	...	...	0.379	0.185	0.495
d_y	in.	18.75	20.45	40.72	18.00	21.18	14.40
b_y	in.	9.10	9.21	10.42	9.30	9.27	9.54
t_2	in.	0.810	0.806	0.787	0.918	0.815	1.086
W_i	psf	57.1	58.1	72.6	54.8	58.2	58.9
β		0.122	0.124	0.155	0.117	0.124	0.126
b) Collapse depth = 4000 ft							
R/t		133.3	120	90	180	120	240
t	in.	1.35	1.50	2.00	1.00	1.50	0.75
σ_y	psi × 10^3	237	214	160	320	214	427
d_x	in.	...	...	...	11.08	26.25	6.63
b_x	in.	...	...	...	2.23	1.68	2.05
t_1	in.	...	...	...	0.574	0.272	0.662
d_y	in.	22.21	26.81	43.13	20.00	27.69	16.36
b_y	in.	11.25	11.54	12.37	11.57	11.62	11.79
t_2	in.	1.178	1.168	1.133	1.365	1.180	1.522
W_i	psf	82.5	85.0	98.7	80.4	85.1	86.6
β		0.176	0.181	0.211	0.172	0.181	0.185
c) Collapse depth = 7000 ft							
R/t		102.9	90	60	120	90	180
t	in.	1.75	2.00	3.00	1.50	2.00	1.00
σ_y	psi × 10^3	320	280	187	374	280	560
d_x	in.	...	...	...	16.94	29.83	7.73
b_x	in.	...	...	...	2.56	2.26	2.52
t_1	in.	...	...	...	0.64	0.432	0.931
d_y	in.	24.84	31.93	63.45	24.00	32.73	18.95
b_y	in.	13.42	13.80	15.13	13.65	13.85	14.23
t_2	in.	1.608	1.589	1.514	1.746	1.605	2.074
W_i	psf	110.4	113.6	142.8	109.2	114.4	120.1
β		0.236	0.242	0.304	0.233	0.244	0.256

[a] Constant data: $E = 29 \times 10^6$ psi, $\rho = 0.3$ lb/in.^{3}, $R = 15$ ft, $L = 30$ ft.

Table 3 Sample elastically stressed titanium hull designs at three ocean depths[a]

Configuration case		Rings only 1	2	3	Rings + stringers 4	5	6
a) Collapse depth = 2000 ft							
R/t		138.5	120	90	180	120	240
t	in.	1.30	1.50	2.00	1.00	1.50	0.75
σ_y	psi $\times 10^3$	123	107	80	160	107	214
d_x	in.	...	...	...	11.95	26.28	6.97
b_x	in.	...	...	...	2.14	1.93	2.10
t_1	in.	...	...	...	0.511	0.313	0.646
d_y	in.	22.22	28.74	46.64	20.00	30.00	17.14
b_y	in.	10.96	11.36	12.23	11.16	11.46	11.44
t_2	in.	1.116	1.105	1.073	1.264	1.120	1.431
W_i	psf	41.8	43.8	51.7	40.5	44.1	42.7
β		0.089	0.093	0.110	0.086	0.094	0.091
b) Collapse depth = 4000 ft							
R/t		100	90	60	120	90	180
t	in.	1.80	2.00	3.00	1.50	2.00	1.00
σ_y	psi $\times 10^3$	178	160	107	214	160	320
d_x	in.	...	...	...	16.41	28.59	7.57
b_x	in.	...	...	...	2.62	2.55	2.54
t_1	in.	...	...	...	0.684	0.508	0.962
d_y	in.	25.52	31.19	61.20	24.00	32.73	18.95
b_y	in.	13.55	13.89	15.16	13.82	14.01	14.44
t_2	in.	1.634	1.621	1.541	1.794	1.650	2.13[illegible]
W_i	psf	60.0	61.3	76.5	59.4	61.9	66.0
β		0.128	0.131	0.163	0.127	0.132	0.141
c) Collapse depth = 7000 ft							
R/t		75	60	45	90	60	120
t	in.	2.40	3.00	4.00	2.00	3.00	1.50
σ_y	psi $\times 10^3$	234	187	140	280	187	374
d_x	in.	...	...	...	19.26	40.09	11.04
b_x	in.	...	...	...	3.17	2.08	3.03
t_1	in.	...	...	...	0.945	0.441	1.179
d_y	in.	29.90	45.86	75.60	27.69	45.00	22.50
b_y	in.	16.21	17.04	18.00[b]	16.57	16.94	17.04
t_2	in.	2.220	2.180	2.128	2.436	2.175	2.729
W_i	psf	80.6	85.5	101.4	80.6	86.1	86.6
β		0.172	0.182	0.216	0.172	0.184	0.185

[a] Constant data: $E = 16 \times 10^6$ psi, $\rho = 0.16$ lb/in.^{3}, $R = 15$ ft, $L = 30$ ft.
[b] Maximum permitted.

weight on the buoyancy factor β is shown by the scale along the right-hand edge of the figure. Here β represents the structural weight of a hull with a length-to-radius ratio of 30 divided by the weight of displaced sea water, where the weights of internal bulkheads (at $L/R = 2$) are ignored, but hemispherical ends having the same W_1/R as the cylindrical hull are included. Elastically stressed designs for a collapse depth of 10,000 ft may have β as low as 0.30; but if the allowable steel yield stress is limited to 150,000 psi, β is sharply increased to 0.50. Note that these depths are collapse depths and not operating depths.

Results for titanium which parallel those discussed previously for steel are presented in Fig. 3. These curves are similar to those in Fig. 2 except all are considerably lower; i.e., titanium elastically stressed designs are lighter than steel ones at any given collapse depth. Again, the yield stress is a significant factor, but because of the tendency for the optimum titanium designs to require thicker hull walls for constant depth and R, the stress induced in the hull wall at a given collapse depth is not as high as in the case of steel. The figure shows that a titanium alloy with a yield stress of about 200,000 psi is almost sufficient to realize the full potential of the ring/stringer configuration at a collapse depth of 10,000 ft. The increased upward sweep to the right in the $R/t = 60$ and 45 curves is caused by the restraint on maximum ring height, which is limiting the efficiency of the designs.

Although Figs. 2 and 3 present weight and wall thickness information (as a function of R), they do not indicate the detail proportions required to yield these weights. Table 2 shows some sample elastically stressed, steel designs for three collapse depths, namely, 2000, 4000, and 7000 ft, in parts a–c, respectively. For each collapse depth, three designs (three R/t's) stiffened only with rings (cases 1–3), and three designs stiffened with both rings and stringers (cases 4–6) are shown. These designs relate to a hull compartment with $R = 15$ ft, $L = 30$ ft; β is as previously noted. Cases 1 and 4 present optimum designs. For the ring/stringer designs (case 4), optimum R/t ratios have been taken directly from Fig. 2; however, the optimum R/t ratios shown for the designs stiffened only with rings (case 1) have been obtained by investigating a range of wall thicknesses in 0.05-in. increments. At all three depths, there is a very slight weight (buoyancy) advantage for the ring/stringer designs. On the other hand, the wall stresses in the ring/stringer designs are significantly higher because of consistently thinner wall thicknesses. It is interesting to note that the ring spacings, and even the ring sizes, are about the same in cases 1 and 4 at any given depth; thus, the stringer-wall geometries in the case 4 designs are roughly equivalent weightwise to the walls between rings in the case 1 designs. Cases 2 and 5 present a comparison of ring and ring/stringer designs for constant R/t and nearly constant weight (buoyancy). The R/t's for these cases represent off-optimum designs for both configurations (i.e., they do not fall on the lower envelope of the curves for each configuration in Fig. 2). Here it is interesting to note that the small differences in weight (buoyancy) at a given collapse depth favor the designs stiffened only with rings (case 2). Wall stresses are the same for both configurations at a given collapse depth, since wall thicknesses are identical. The rather small amount of weight allocated to the stringers

Table 4 Sample elastically stressed designs showing greater depth capability of ring/stringer-stiffened hulls for constant hull weight and R/t

Configuration case		Rings only 1	2	Rings + stringers 3
a) Steel designs[a]				
t	in.	1.14	1.00	1.00
d_x	in.	...	...	13.09
b_x	in.	...	...	2.16
t_1	in.	...	...	0.472
d_y	in.	20.12	17.91	21.18
b_y	in.	10.217	11.43	10.69
t_2	in.	1.000	1.029	1.140
Collapse depth	ft	2950	2440	3100
σ_y	psi $\times 10^3$	208	195	248
b) Titanium designs[b]				
t	in.	1.75	1.50	1.50
d_x	in.	...	...	16.99
b_x	in.	...	...	2.56
t_1	in.	...	...	0.646
d_y	in.	25.48	22.12	24.00
b_y	in.	13.31	14.92	13.64
t_2	in.	1.579	1.612	1.742
Collapse depth	ft	3750	3025	3850
σ_y	psi $\times 10^3$	172	162	206

[a] $R = 15$ ft, $L = 30$ ft, $E = 29 \times 10^6$ psi, $\rho = 0.30$ lb/in.^{3}, $W_i = 70.0$ psf, $\beta = 0.149$.
[b] $R = 15$ ft, $L = 30$ ft, $E = 16 \times 10^6$ psi, $\rho = 0.16$ lb/in.^{3}, $W_i = 58.0$ psf, $\beta = 0.124$.

in the case 5 designs is seen to be obtained by spacing the rings further apart than in the case 2 designs. The case 3 designs have been included to show the effect upon detail geometry and weight of reducing wall stress through increases in wall thickness. The designs for case 6, which have the highest stresses for a given collapse depth, are included primarily to show that for designs stiffened with rings and stringers, the one with the highest stress is not necessarily the lightest (the designs for case 4 are lightest for a given collapse depth). The stringers in all of the designs stiffened with rings and stringers tend to be relatively small in size compared to the rings. Neither the stringers or rings are limited by the restraint imposed on height; also, the slenderness ratios for both rings and stringers are less than the maximum value of this restraint. Therefore, the height-thicknesses proportions of these elements are being governed by local instability requirements. Finally, although several of the designs shown in Table 2 appear to be impractical from the standpoint of the requirement for very high yield strengths, they do indicate the properties that must be available in steel in order to take full advantage of the structural potential of these configurations at the collapse depths investigated.

Table 3 presents sample designs for titanium hulls which parallel the designs for steel hulls shown in Table 2. The titanium designs show a significant weight advantage over their steel counterparts; other observations of Table 3 parallel those discussed previously with reference to Table 2. Note, however, that the titanium rings are deeper than the steel rings for comparable designs, and that the maximum ring height is reached in one of the designs for 7000-ft collapse depth.

In contrast to the designs developed in Tables 2 and 3 for constant collapse depths, Table 4 presents some sample designs developed for constant design weight. Here, an optimum design stiffened only with rings (case 1) is compared to an optimum design stiffened with rings and stringers (case 3) to show relative collapse depth capabilities; two steel designs are compared in part a of Table 4, and two titanium designs are compared in part b. Again, the optimum R/t ratios for the ring/stringer designs (case 3) have been taken directly from Figs. 2 and 3, whereas the optimum R/t ratios for the designs stiffened only with rings (case 1) are the result of a detailed study of various wall thicknesses. The ring/stringer designs (case 3) are seen to have a slightly greater collapse depth capability than the designs stiffened only with rings (case 1). The wall thicknesses in the ring/stringer designs are thinner here also, and wall stresses are again significantly higher than those shown for the case 1 designs. The case 2 designs are included to show a comparison between ring/stringer designs and designs stiffened only with rings when wall thickness is held constant. In connection with this comparison, the constant weights have been so chosen to show a definite variation in the collapse depth capabilities of the two configurations (see Figs. 2 and 3). Comparison of the case 2 and 3 designs shows that there are regions where rather large differences in depth capabilities exist if wall thickness is arbitrarily held constant. Of further interest in Table 4 is that ring spacing and size are approximately the same for case 2 and 3 designs in both parts a and b. Thus, it would appear possible, although not necessarily feasible, to increase the collapse depth of a hull originally stiffened only with rings by adding stringers and accepting a modest weight penalty through the use of the existing, slightly nonoptimum rings. Of course, in order to utilize stringers in this manner in submersibles, the material in the hull wall must have a sufficient reserve strength to elastically support the additional stress induced in the hull wall by the higher pressure at a deeper collapse depth. This seems rather unlikely. In most existing designs, as previously noted, strength is often an overriding consideration in ring-stiffened hulls even at relatively shallow collapse depths. In passing, it should be emphasized that the data in Tables 2–4 have been obtained from a nondimensional analysis. If $R = 7.5$ ft had been chosen, rather than $R = 15.0$ ft, the same R/t's would be studied, but because R has been halved, t is halved, and σ_y remains constant. The remaining dimensions would be scaled also.

Conclusions

Cylindrical pressure hulls stiffened with rings and stringers and capable of sustaining hydrostatic collapse pressures encountered at ocean depths of 1000 ft or more have a structural efficiency which is ideally only very slightly superior to that for hulls stiffened with rings only. Although hull walls stiffened with rings and stringers may be preferred because they tend to be thinner at a given collapse depth and therefore are easier to join by welding, the prospects for taking advantage of this fact are slim because the associated higher wall stresses, even at relatively shallow ocean depths, often exceed allowable yield stresses in the steel and titanium alloys now advisable for large pressure hull construction. For the same reason, it is unlikely that stringers will be used to extend the collapse depth capabilities of many existing hulls originally stiffened only with rings. This being the case, ring cross sections which are more efficient (Ref. 3) than the one studied here obviously cannot be effectively utilized until alloys with much higher yield strengths after welding, in combination with other necessary properties, are developed. It is clear that low β factors (or high structural efficiencies) for future large cylindrical pressure hulls fabricated from either steel or titanium for commercial or tactical submersibles depend primarily upon significant advances in the metallurgy of these materials and not on the choice of configuration.

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